Math 215, Winter 2014
Midterm 2, February 27

Name: **SID:**

Instructor: **Section:**

Instructions

- The total time allowed is 60 minutes.
- The total score is 50 points.
- Use the reverse side of each page if you need extra space.
- Show all your work. A correct answer without intermediate steps will receive no credit. However, in question 2, intermediate steps do not need to be shown.
- Calculators, phones and cheat sheets are not allowed.

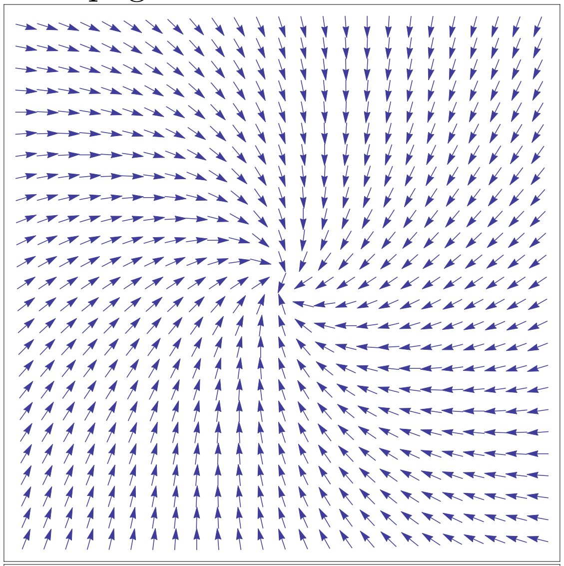
Problem	Points	Score
1	15	
2	15	
3	10	
4	10	
TOTAL	50	

1. Consider the following system of differential equations with initial conditions.

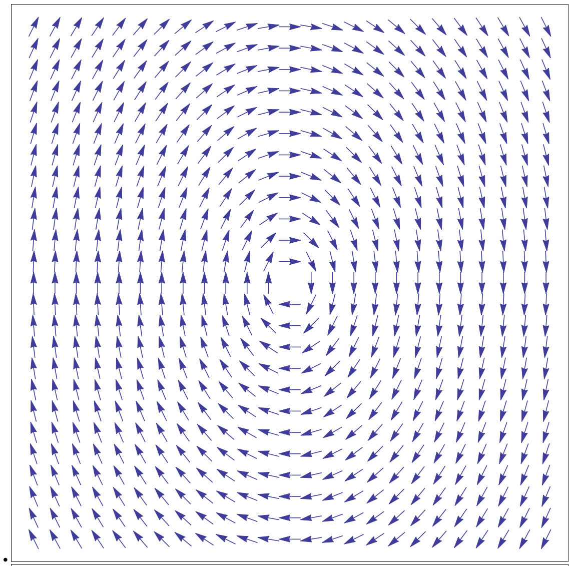
$$\mathbf{x}'(t) = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} \mathbf{x}(t) \qquad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Solve the problem and fully simplify your answer.

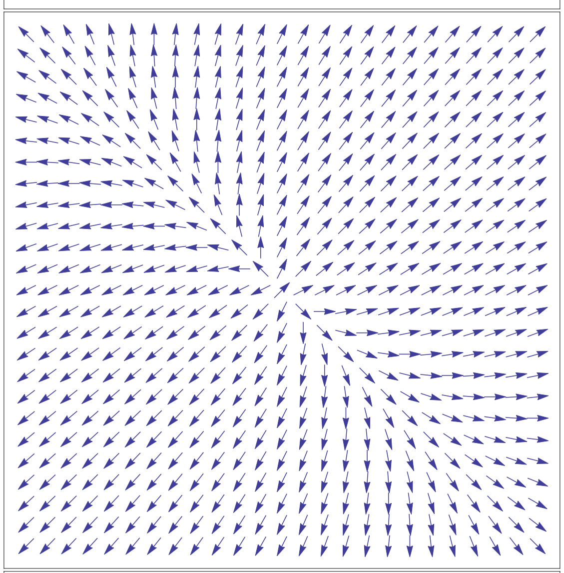
2. No part marks will be awarded for this question. Match the equations on the next page to these direction fields.



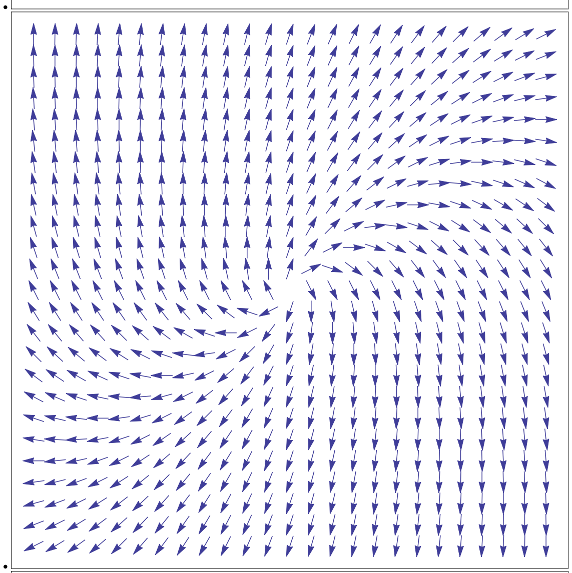
1.



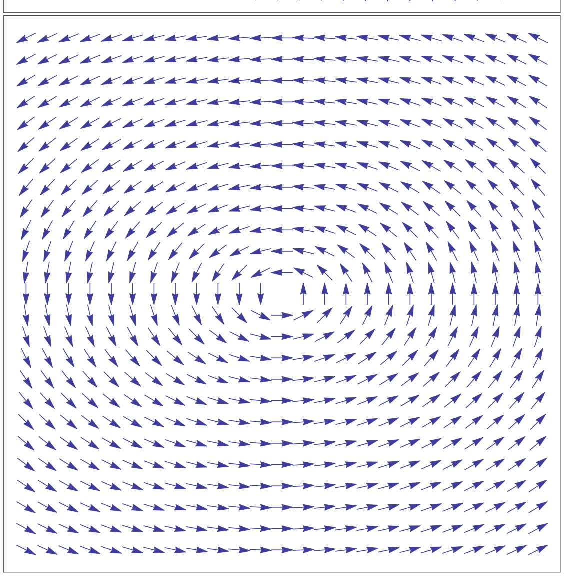
2.



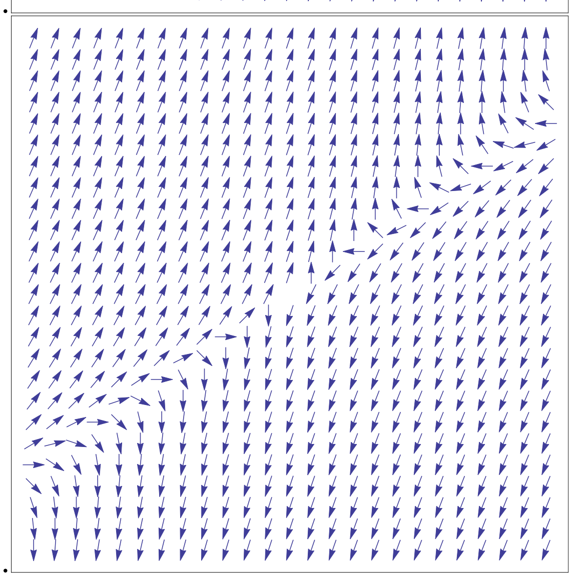
3.



4.



5.



6.

Solution (fill in your answer here):

$$\left|(A) \quad \mathbf{x}' = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{x}\right| \qquad \text{matches direction field}$$

$$\left|(B) \quad \mathbf{x}' = \begin{pmatrix} 1 & 1 \\ -2 & 3 \end{pmatrix} \mathbf{x}\right| \qquad \text{matches direction field}$$

$$\left|(C) \quad \mathbf{x}' = \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix} \mathbf{x}\right| \qquad \text{matches direction field}$$

$$\left|(D) \quad \mathbf{x}' = \begin{pmatrix} -1 & 1 \\ -2 & 3 \end{pmatrix} \mathbf{x}\right| \qquad \text{matches direction field}$$

$$\left|(E) \quad \mathbf{x}' = \begin{pmatrix} -3 & 1 \\ -2 & -3 \end{pmatrix} \mathbf{x}\right| \qquad \text{matches direction field}$$

3. A stylized model of competing species with population densities $x(t)$ and $y(t)$ is given by

$$\frac{dx}{dt} = x\left(\frac{3}{2} - x - \frac{1}{2}y\right),$$
$$\frac{dy}{dt} = y\left(2 - y - \frac{3}{4}x\right).$$

Find and classify all coexistence critical points (critical points where both species are present).

4. Consider the nonlinear system

$$\mathbf{x}'(t) = \begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} 2y - \beta x^2 \\ -x - \beta xy \end{pmatrix}$$

where β is a real parameter.

a. Find all (real) critical point(s) of the system.

b. Calculate the Jacobian of the system at the critical point(s).

c. Classify the critical point(s).

d. Find a value of β so that all solutions of the full system are periodic. Justify your answer.