Math 215, Winter 2014
Midterm 1, January 30

Name: **SID:**

Instructor: **Section:**

Instructions

- The total time allowed is 60 minutes.
- The total score is 50 points.
- Use the reverse side of each page if you need extra space.
- Show all your work. A correct answer without intermediate steps will receive no credit.
- Calculators, phones and cheat sheets are not allowed.

Problem	Points	Score
1	18	
2	12	
3	10	
4	10	
TOTAL	50	

1. Solve the following differential equations for $y(t)$:

a. $\frac{dy}{dt} = 6t^2 - \frac{y}{t}$, with $y(1) = 1$.

Solution:

b. $\frac{dy}{dt} = (y - y^2)te^{t^2}$, with $y(0) = 2$.

Solution:

2. Match the direction fields with the differential equations by circling A, B, C, or D in each case. Some of these differential equations do not match any of the direction fields - for those cases circle "none".

(1) $y' = y^2(2-y)$ A B C D none

(2) $y' = y(2-y)$ A B C D none

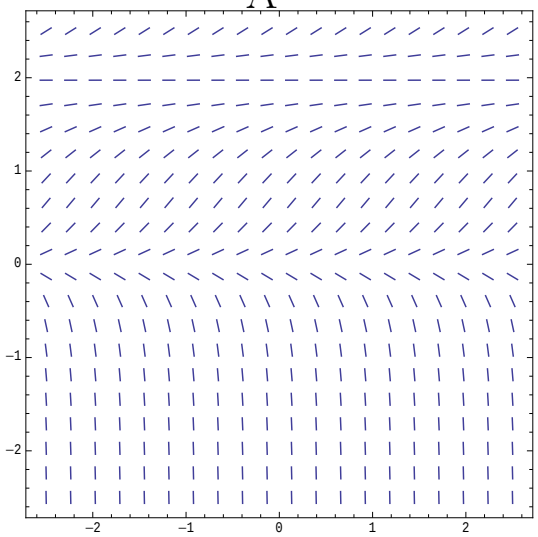
(3) $y' = y(2-y)^2$ A B C D none

(4) $y' = y - 2x$ A B C D none

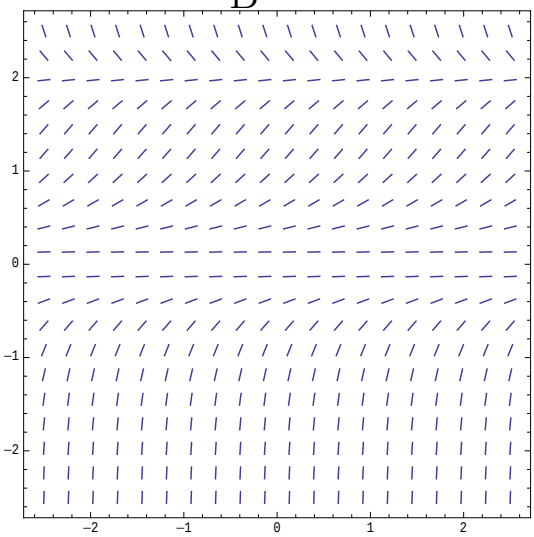
(5) $y' = y - \sin x$ A B C D none

(6) $y' = y + 2x$ A B C D none

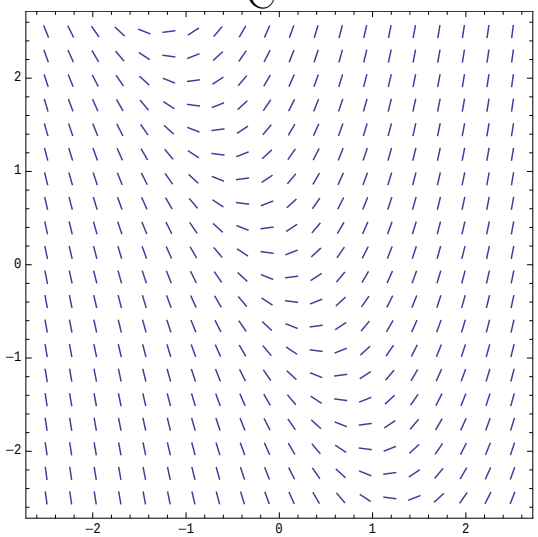
A



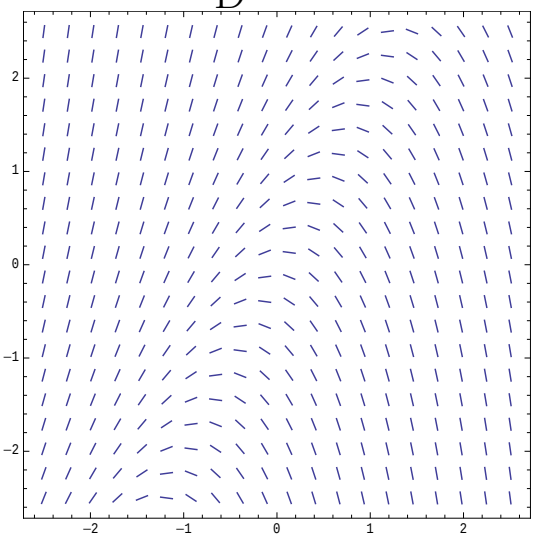
B



C



D



3. Consider the following system of differential equations with initial conditions.

$$\mathbf{x}'(t) = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \mathbf{x}(t) \qquad \mathbf{x}(0) = \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix}$$

a. Solve the system for $x_1(0) = 1, x_2(0) = 0$.

Solution:

3 b. Find all possible initial conditions for the system so that $|\mathbf{x}(t)|$ does not go to infinity as $t \to \infty$.

4. For this question, assume that turkeys follow Newton's law of cooling/heating. I start my oven heating at 12pm. The temperature of the oven increases linearly with time until it reaches 150°C at 12.30pm. The oven is initially at room temperature, 20°C.

a. Write an equation for the temperature of the oven as a function of t, the time in hours since 12pm.

b. A turkey is placed in the oven at 12pm. The relaxation time of the turkey is 1 hour. (Hint: this means that the constant in Newton's law is $\pm$ 1/hour depending on your sign convention). The temperature of the turkey at 12pm is 4°C. Find the temperature of the turkey at 12.30pm (leave powers of e as part of your answer).