**The University of British Columbia**

Midterm Examination (Version 1) - October 4, 2012

Closed book examination Time: 50 minutes

**Last Name** _____ **First** _____

**Student Number** _____ **Signature** _____

**MATH 104 or MATH 184** (Circle one) **Section Number:** _____

**Special Instructions:**

No memory aids, calculators, or electronic devices of any kind are allowed on the test. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. Numerical answers should be left in calculator-ready form, unless otherwise indicated. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

**Rules governing examinations**

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

| | | |
|---|---|---|
| 1 | | 10 |
| 2 | | 7 |
| 3 | | 8 |
| 4 | | 8 |
| 5 | | 8 |
| 6 | | 9 |
| Total | | 50 |

[10] **1**.
Compute the following limits:

a) (2 marks) $\lim_{x\to 3} \dfrac{x^2-9}{x^2-8x+15}$

b) (3 marks) $\lim_{x\to 1} \dfrac{x-1}{\sqrt{2x-1}-1}$

Compute the derivatives of the following functions.

c) (2 marks) Find $f'(x)$ where $f(x) = (x^2 + 7x)(e^x + x^3 + 2x^2 + 1)$.
DO NOT SIMPLIFY YOUR ANSWER.

d) (3 marks) Find $h'(1)$ where $h(x) = \dfrac{xf(x) - 5}{g(x)}$, $f'(1) = 2$, $g'(1) = -3$ and $f(1) = 1$, $g(1) = 1$. EXPRESS YOUR ANSWER AS AN INTEGER.

[7] **2**. Let $f(x)$ be a function defined for all $x$ near some number $a$.

(a) (2 marks) Carefully state the definition of $f'(a)$, the derivative of $f(x)$ at the point $x = a$.

(b) (5 marks) Suppose $f(x) = \dfrac{2}{x-1}$. Show that $f'(a) = \dfrac{-2}{(a-1)^2}$ using the definition of the derivative. NO credit will be given for any other method.

[8] **3**. A spaceship travels along a path given by the graph $y = x^2 \sin(x)$ in the plane.

a) (2 marks) What is the $y$ coordinate of the ship when $x = \pi$?

b) (6 marks) At the point in part (a), a piece breaks off of the ship and travels along the tangent line to the ship's path at this point. What is the $y$ coordinate of this piece when its $x$ coordinate is $x = 2\pi$?

[8] **4**. ABC Inc. has recently introduced the ABC smartphone. They anticipate that if they sell the smartphone at the price of \$300 per unit, they will sell 5000 units per week. For each \$10 increase in the price, they anticipate selling 200 fewer units per week. The fixed costs of producing the smartphone are \$100000 per week, and each smartphone costs ABC \$50 to make.

a) (2 marks) Let $p$ be price and $q$ be weekly demand for the smartphone. Find the linear demand function $p(q)$.

b) (1 marks) Find the weekly cost function $C(q)$.

c) (2 marks) The weekly profit function $P(q)$ is given by $P(q) = -\frac{1}{20}q^2 + 500q - 100000$. Find the marginal weekly profit function $MP(q)$ (The Marginal Profit function is just the derivative of the profit function: $MP(q) = P'(q)$).

d) Suppose that the price is currently $200 per unit. If the price is increased by a small amount,

i) (1 marks) Will the quantity demanded increase or decrease? (explain)

ii) (2 marks) Will the weekly profit increase or decrease? (explain)

[8] **5**. Consider the function

$$f(x) = \begin{cases} ax^2 + 1 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ bx^3 + cx & \text{if } x > 1. \end{cases}$$

(a) (2 marks) Compute the left and right hand limits of $f(x)$ as $x \to 1$.

(b) (2 marks) What equations must $a, b$ and $c$ satisfy so that $f$ is continuous at $x = 1$?

(c) (4 marks) What equations must $a, b$ and $c$ satisfy for $f$ to be differentiable at $x = 1$? Determine the values of $a, b, c$ for which $f$ will be differentiable at $x = 1$.

[9] **6**. For each question below, either explain why the statement is true or show the statement is false by providing a counter example if appropriate. No credit will be given for answers without justification.

(a) The equation $\dfrac{x^3 - 2\sqrt{x} - 5}{e^x} = 0$ has a solution.

(b) The following function is continuous at $x = 3$

$$f(x) = \begin{cases} \dfrac{x^2+1}{x-1}, & \text{if } x \neq 3, \\ 5, & \text{if } x = 3, \end{cases}$$

(c) If $f(x) + g(x)$ is differentiable at $x = 1$, then both $f(x)$ and $g(x)$ must also be differentiable at $x = 1$.

**The End**