This examination has 6 pages of questions <u>excluding</u> this cover

The University of British Columbia
Midterm 2 - March 16, 2011

Mathematics 103: Integral Calculus with Applications to Life Sciences

201 (Holmes), 203 (Hauert), 206 (Rolfsen), 207 (Christou), 208 (Lindstrom), 209 (Rolfsen)

Closed book examination Time: 60 minutes

Last Name: ______________________ **First Name:** ______________________

Student Number: ________________ **Section:** circle above

Rules governing examinations:

1. No books, notes, electronic devices or any papers are allowed. To do your scratch work, use the back of the examination booklet. Additional paper is available upon request.
2. You should be prepared to produce your library/AMS card upon request.
3. No student shall be permitted to enter the examination room after 10 minutes or to leave before the completion of the examination.
4. You are not allowed to communicate with other students during the examination. Students may not purposely view other's written work nor purposefully expose his/her own work to the view of others or any imaging device.
5. At the end of the exam, you will put away all writing implements and calculators upon instruction. Students will continue to follow all of the above rules while the papers are being collected.
6. Students must follow all instructions provided by the invigilator.
7. Students are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions
8. Any deviation from these rules will be treated as an academic misconduct. The plea of accident or forgetfulness shall not be received.

I agree to follow the rules outlined above ________________________________
(signature)

Question:	1	2	3	4	5	Total
Points:	10	10	9	9	12	50
Score:						

Show all your work and explain your reasonings clearly!

1. (10 points) Short Answer Problems (show your work).

   a. (3) Consider the function $f(x) = \int_1^x \sqrt{t^3 - 1}dt$ for $x \geq 1$. Calculate the length of the curve $y = f(x)$ from $x = 1$ to $x = 4$.

   ANSWER: $L =$ ____________________

   b. (4) A gum ball machine dispenses red, blue and green gum balls. Half of the gum balls are red and there are twice as many blue gum balls than green ones. What is the probability to get *at least two* blue gum balls in four trials? (Note that there are many gum balls so that the probabilities to obtain a particular color remain constant. Simplify your result as much as possible but no decimals.)

   ANSWER:____________________

   c. (3) Evaluate the integral $I = \displaystyle\int \frac{1}{x^2 + 2x + 2} dx$.

   ANSWER: $I =$ ____________________

2. (10 points) Evaluate the integrals:

a. (4)

$$I_1 = \int_0^{\frac{3\pi}{2}} \cos(t) e^{2\sin(t)} dt$$

ANSWER: $I_1 =$ ______________________

b. (6)

$$I_2 = \int \frac{x-1}{x^2 - x - 2} dx$$

ANSWER: $I_2 =$ ______________________

3. (9 points) A bar of a special aluminum alloy of length $L = 1/2$ meters has a variable linear density $\rho(x) = \cos(\pi x)$ (in kg/m, with x the distance in meters along the bar starting at the left side).

   a. (3) Calculate the total mass of the bar.

   ANSWER: $M =$ ______________________

   b. (2) Calculate the average density.

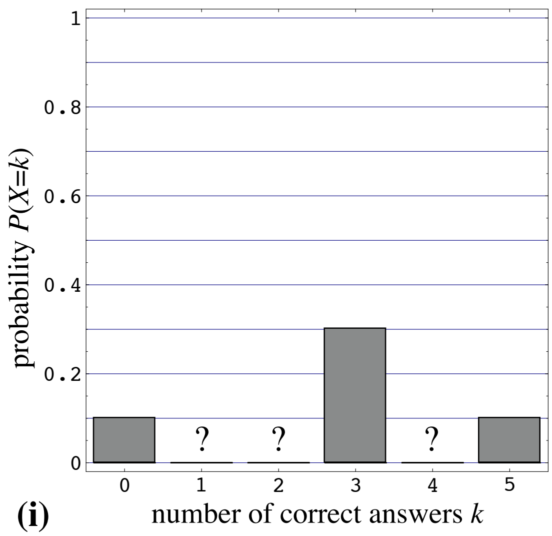
   ANSWER: $\bar{\rho} =$ ______________________

   c. (4) Calculate the center of mass.

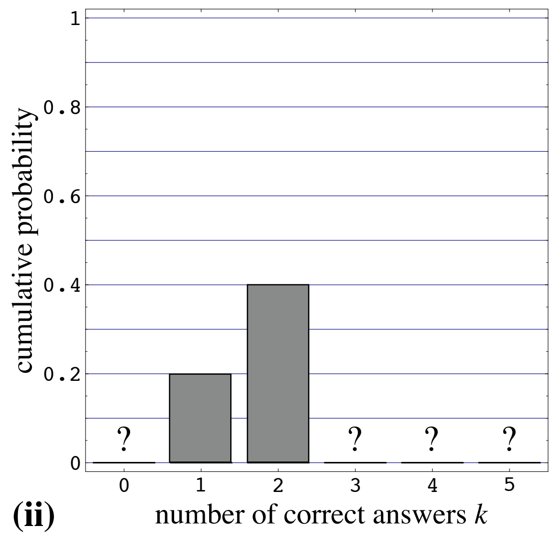
   ANSWER: $\bar{x} =$ ______________________

4. (9 points) **Part I:** Students write a quiz with five questions. Some probabilities for having k answers correct are shown in diagram (i) and additional information is provided as cumulative probabilities in diagram (ii).

   a. (5) Complete the two diagrams for the probability distribution (i) and the cumulative probability distribution (ii) by combining the information in both diagrams:



(i)



(ii)

   b. (2) Calculate the probability that at most four questions were correctly answered, $P(X \leq 4)$.

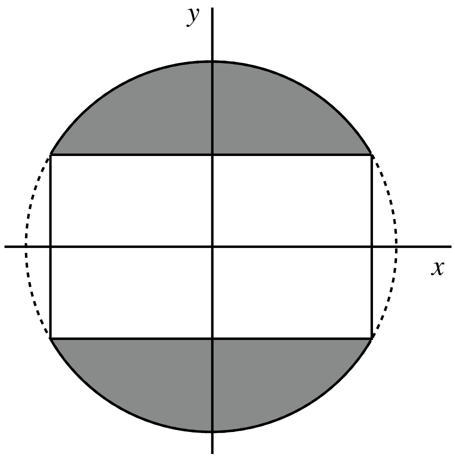
   ANSWER: $P(X \leq 4) =$ ____________________

Part II: (2) Consider a lottery with tickets for \$10. The possible winnings are \$5, \$100, \$500 and \$10,000 with probabilities 0.1, 0.02, 0.001 and 0.0001 respectively. What are your expected winnings from a single ticket?

ANSWER: ____________________

5. (12 points) A spherical bead is made by drilling a hole of radius $1cm$ right through the centre of a wooden sphere of radius $2cm$. What is the volume of that bead?

   a. (8) Setup and evaluate an integral to calculate its volume V. (In order to assist in setting up the integral, the shaded region in the sketch below depicts a cross section of the bead through the centre of the hole that was drilled along the x-axis.)



   ANSWER: $V =$ ____________________

   b. (4*) Find the area A of the cross section of the bead through its center, i.e. the shaded area in the sketch above. (Note that this part is more challenging - use back of page for your notes.)

   ANSWER: $A =$ ____________________

Useful Formulæ

Summation

$$\sum_{k=1}^{N} k = \frac{N(N+1)}{2}$$

$$\sum_{k=1}^{N} k^2 = \frac{N(N+1)(2N+1)}{6}$$

$$\sum_{k=1}^{N} k^3 = \left(\frac{N(N+1)}{2}\right)^2$$

$$\sum_{k=0}^{N} r^k = \frac{1-r^{N+1}}{1-r}$$

Trigonometric identities

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta; \qquad \text{for } \alpha=\beta\text{:} \quad \sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta; \qquad \text{for } \alpha=\beta\text{:} \quad \cos(2\alpha) = 2\cos^2\alpha - 1$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\tan^2\alpha + 1 = \sec^2\alpha = \frac{1}{\cos^2\alpha}$$

Some useful trigonometric values

$$\sin(0) = 0, \quad \sin(\frac{\pi}{6}) = \frac{1}{2}, \quad \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}, \quad \sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}, \quad \sin(\frac{\pi}{2}) = 1, \quad \sin(\pi) = 0$$

$$\cos(0) = 1, \quad \cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}, \quad \cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}, \quad \cos(\frac{\pi}{3}) = \frac{1}{2}, \quad \cos(\frac{\pi}{2}) = 0, \quad \cos(\pi) = -1$$

Derivatives

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$