

This examination has 6 pages of questions excluding this cover

The University of British Columbia

Midterm 1 - February 7, 2011

Mathematics 103: Integral Calculus with Applications to Life Sciences

201 (Holmes), 203 (Hauert), 206 (Rolfsen), 207 (Christou), 208 (Lindstrom), 209 (Rolfsen)

Closed book examination

Time: 60 minutes

Last Name: _____

First Name: _____

Student Number: _____

Section: circle above

Rules governing examinations:

1. No books, notes, electronic devices or any papers are allowed. To do your scratch work, use the back of the examination booklet. Additional paper is available upon request.
2. You should be prepared to produce your library/AMS card upon request.
3. No student shall be permitted to enter the examination room after 10 minutes or to leave before the completion of the examination.
4. You are not allowed to communicate with other students during the examination. Students may not purposely view other's written work nor purposefully expose his/her own work to the view of others or any imaging device.
5. At the end of the exam, you will put away all writing implements and calculators upon instruction. Students will continue to follow all of the above rules while the papers are being collected.
6. Students must follow all instructions provided by the invigilator.
7. Students are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions
8. Any deviation from these rules will be treated as an academic misconduct. The plea of accident or forgetfulness shall not be received.

I agree to follow the rules outlined above _____
(signature)

Question:	1	2	3	4	5	Total
Points:	18	6	10	6	10	50
Score:						

Show all your work and explain your reasonings clearly!

Some useful formulas:

$$\sum_{k=1}^N k = \frac{N(N+1)}{2}, \quad \sum_{k=1}^N k^2 = \frac{N(N+1)(2N+1)}{6},$$
$$\sum_{k=1}^N k^3 = \frac{N^2(N+1)^2}{4}, \quad \sum_{k=0}^N r^k = \frac{1-r^{N+1}}{1-r} \quad (r \neq 1)$$

1. (18 points) Short Answer Problems (only the final answer is marked)

a. Evaluate the sum: $\sum_{k=3}^{10} (1 + k^2)$.

ANSWER:_____

- b. Suppose an ant runs a distance D_0 then stops. After a brief rest it runs two thirds of the distance $D_1 = 2D_0/3$ before taking another brief rest. If this alternation of running and resting continues forever, each time running $2/3$ the distance of its previous run, how far does the ant get assuming $D_0 = 16 \text{ cm}$?

ANSWER:_____

c. Evaluate the integral: $\int_0^T (1 - e^{3x})dx$.

ANSWER:_____

- d. Find the average of the function $f(x) = 1 + \sin(\pi x)$ for $1 \leq x \leq 4$.

ANSWER:_____

- e. Consider the function $B(x) = \int_x^1 \frac{1}{1+t^4} dt$. Calculate its derivative $B'(x)$.

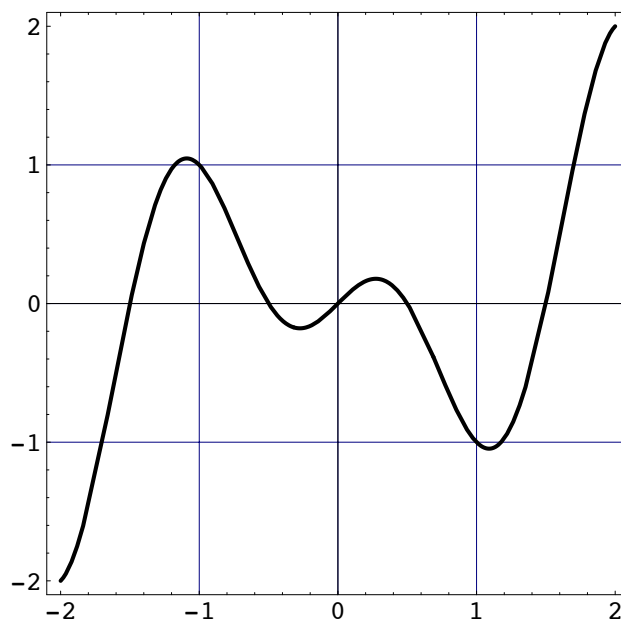
ANSWER:_____

- f. Consider the Riemann sum $\lim_{N \rightarrow \infty} \frac{1}{N^5} \sum_{k=1}^N k^4$. Rewrite the sum as a definite integral and evaluate it (do not attempt to evaluate the sum directly).

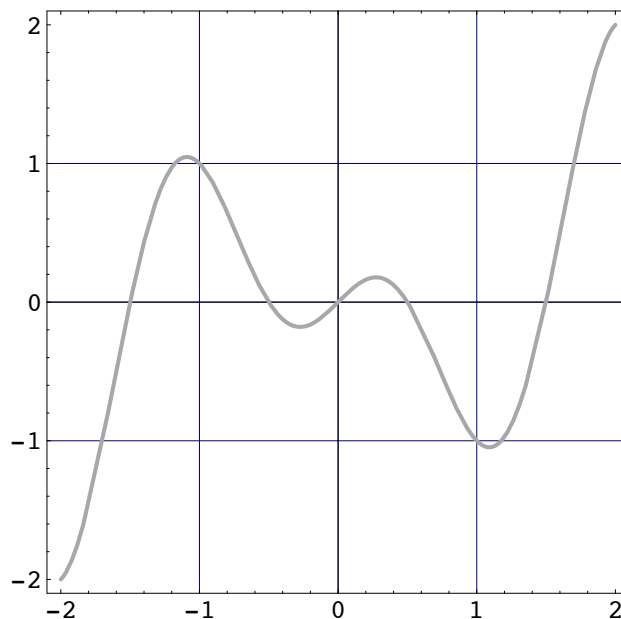
ANSWER:_____

2. (6 points) Consider the function $f(x)$ shown in the figure below and the related function $F(x) = \int_{-1}^x f(s) ds$ for $-2 \leq x \leq 2$.

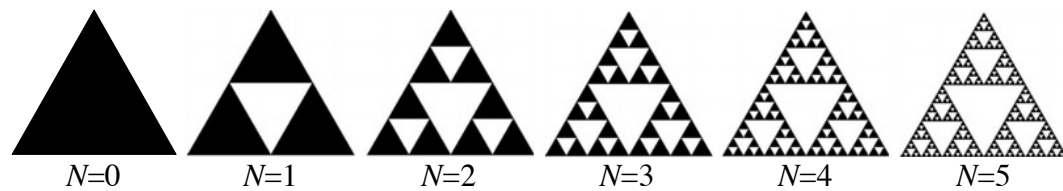
- Mark and label the x value(s) where $F(x)$ has a relative maximum or minimum.
- Mark and label the x value(s) where $F(x)$ has an inflection point.
- Mark the interval(s) of x for which $F(x) \geq 0$.



- Sketch the graph of $F(x)$.



3. (10 points) The Sierpinski triangle is formed by taking an equilateral triangle and recursively removing equilateral triangles as shown in the figure.



- a. If we begin with an equilateral triangle of area 1, what is the area of the shaded region after N steps of removing triangles?
- b. What is the limit of the shaded area as $N \rightarrow \infty$? (justify your answer)
- c. If we begin with an equilateral triangle with a side length of 1, what is the length of the perimeter of the shaded region after N steps?
- d. What is the limit of the perimeter of the shaded region as $N \rightarrow \infty$? (justify your answer)

4. (6 points) Calculate the finite area bounded by the two curves $f(x) = x^2 - 1$ and $g(x) = x(1 - x)$.

5. (10 points) A rollercoaster cart glides along a linear frictionless track. At time t , the position of the cart is $x(t)$, its velocity $v(t)$ and its acceleration $a(t)$.
- a. Suppose that for $0 \leq t \leq 3$ seconds, the cart is pulled by an engine and accelerated such that $a(t) = 3kt^2$ [m/s^2], where k is constant. The initial position is $x(0) = 0$ and after 2 seconds the cart is at $x(2) = 8$ meters. Find $x(t)$ and $v(t)$ for these 3 seconds and express your answer in terms of t and k .
- b. At $t = 3$ the engine stops and the velocity of the cart is $v(3) = 19$ [m/s]. Find the initial velocity $v(0)$ of the cart. (Hint: this is a number.)
- c. For $t > 3$, air resistance slows the cart down according to $a(t) = -r v(t)$ with $r = 1/3$. Compute $v(t)$.
- d. At what time T has the air resistance reduced the speed of the cart by half (to $v(T) = 19/2$ [m/s])?
- e. When will the cart come to a full stop?