Name:

Signature:

Student Number:

Math 101 Section 208   **Test #3 - Monday, March 29, 2010**   **Time: 50 minutes**

Don't worry - just do your best :-)

1. The marks you receive during the term will **likely be scaled** based on the class results on the final exam. Even if it seems difficult, that doesn't mean the term marks will be low!

2. For questions $2-5$, **generous marks** may be given for honest attempts, even if you get stuck - provided you show your work.

Instructions and policies

1. Please print your **name**, **student number**, and provide your **signature** right away, before you attempt any of the problems. **Check** that this booklet contains **eleven (11) pages**.

2. **All your work** must be shown for full credit on questons $2-5$. **No work** is required in question 1, and a correct answer will receive 3 points; partial credit of 1 point may be given in question 1 if the answer you provide is wrong.

3. **Unless** a question **specifies otherwise**, you **do not** need to **simplify** your answers.

4. **No calculators, electronic devices, or formula sheets** are allowed during the test.

5. **Ten (10) minutes before the end of the test** period you will be given a verbal notice. After that time, you must **remain seated** until all test papers have been collected.

6. **Exposing your test paper** to others or **looking** at the paper of another student, even if unintentional, will be deemed as **cheating** and subject to disciplinary action.

7. When the test period is over, you will be instructed to put away writing implements. Put away all pens and pencils at this point. **Continuing to write past this instruction** will be **considered cheating**. Please remain seated and pass your test paper down the row to the nearest indicated aisle. You may leave your seat only after all the tests have been collected.

8. A certain **portion of the tests** will be **randomly selected and photocopied**. This is a warning in case an individual modifies their answers and asks for a re-grading, which would be cheating.

1 Short Answer (15 marks: 3 marks per part)

a Find the length of the curve $r = \sin(\theta)$ from $\theta = 0$ to $\theta = \pi/3$.

b Write down the Trapezoid rule approximation T_4 for evaluating $\int_{-1}^{1} \frac{dx}{2-x^4}$.

c Find the *first three nonzero terms* in the Maclaurin series for $\arcsin(2x)$ in powers of x.

d Evaluate $\int_{-1}^{1} x^3 \sqrt{\frac{1+4x^2}{1+x^4}}\,dx$.

e A tank in the form of a rectangular prism of base area 4 ft^2 and height 10 ft is *completely filled* with water. Approximating the weight of water to be 60 lbs ft^{-3}, find the work done in pumping all the water *out of the top* of the tank. *Simplify* your answer as much as possible.

2 Long Answer (10 marks)

Find the volume of the described solid S : the base of S is an elliptical region with boundary curve $9x^2 + 4y^2 = 36$. Cross-sections perpendicular to the $x-$axis are isosceles right triangles with hypotenuse in the base.

3 Long Answer (10 marks)

Find

    a (5 marks) $\int x^2 \tan^{-1} x\, dx$

    b (5 marks) $\int_0^{\frac{\pi}{4}} \frac{\cos\theta\, d\theta}{(1-\sin\theta)^{2/3}}$

4 Long Answer (10 marks)

In this problem you may (or may not) find the following facts useful:

$$\lim_{x \to \infty} \frac{\ln x}{x^a} = 0 \text{ for } a > 0$$

$$\lim_{x \to 0^+} x^a \ln x = 0 \text{ for } a > 0$$

$$\int \frac{dx}{\sqrt{x^2 + 1}} = \ln|\sqrt{x^2 + 1} + x| + C$$

a (2 marks) Show that $\int f(x) \ln x \, dx = F(x) \ln x - \int \frac{F(x)}{x} dx$ where $F'(x) = f(x)$.

b (4 marks) Show that $\int_1^\infty \frac{\ln x}{x^p} dx$ converges for $p > 1$ and diverges for $p \leq 1$.

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c (2 marks) Does $\int_0^1 \frac{\ln x}{\sqrt{x}} dx$ converge?

d (2 marks) For which values of s does $\int_0^\infty \frac{\ln x}{\sqrt{x^s+1}} dx$ converge?

5 **Conceptual Question** (5 marks)

Consider a smooth, differentiable cartesian curve $y = f(x)$ for $x \in [a, b]$.

a (3 marks) By breaking $[a, b]$ into n subintervals of *equal width*, come up with a Riemann sum approximation to the arc length by approximating f as a straight line over each subinterval.

b (2 marks) Derive the arc length formula from your Riemann sum.
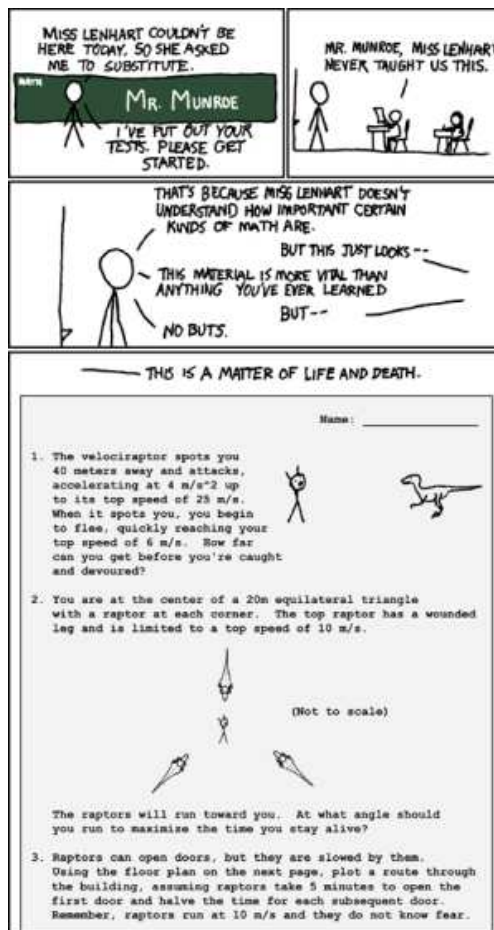
Figure 1: Perhaps the greatest XKCD comic ever - hope you enjoy!

Bonus a (1 mark)

A great philosophical question: why should you get a bonus mark?

Bonus b (1.5 marks)

Evaluate $\int_0^{\pi/2} \frac{d\theta}{1+(\tan\theta)^{\sqrt{2}}}$. Hint: try a substitution with $\psi = \frac{\pi}{2} - \theta$...

This page is provided for rough work or extra space.

Mark breakdown:

Question 1 (out of 15)	————
Question 2 (out of 10)	————
Question 3 (out of 10)	————
Question 4 (out of 10)	————
Question 5 (out of 5)	————
Bonus (out of 2.5)	————
TOTAL (out of 50)	————