Name: _____
Signature: _____
Student Number: _____

Math 101 Section 208 Test # 1 - Monday, February 1, 2010 Time: 50 minutes

Don't worry - just do your best :-)

1. The marks you receive during the term will **likely be scaled** based on the class results on the final exam. Even if it seems difficult, that doesn't mean the term marks will be low!
2. **Generous marks** may be given for honest attempts, even if you get stuck - provided you show your work.

Instructions and policies

1. Please print your **name**, **student number**, and provide your **signature** right away, before you attempt any of the problems. **Check** that this booklet contains **ten (10) pages**.
2. **All your work** must be shown for full credit.
3. **Unless** a question **specifies otherwise**, you **do not** need to **simplify** your answers.
4. **No calculators, electronic devices, or formula sheets** are allowed during the test.
5. **Ten (10) minutes before the end of the test** period you will be given a verbal notice. After that time, you must **remain seated** until all test papers have been collected.
6. **Exposing your test paper** to others or **looking** at the paper of another student, even if unintentional, will be deemed as **cheating** and subject to disciplinary action.
7. When the test period is over, you will be instructed to put away writing implements. Put away all pens and pencils at this point. **Continuing to write past this instruction** will be **considered cheating**. Please remain seated and pass your test paper down the row to the nearest indicated aisle. You may leave your seat only after all the tests have been collected.
8. A certain **portion of the tests** will be **randomly selected and photocopied**. This is a warning in case an individual modifies their answers and asks for a re-grading, which would be cheating.

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1 **Short Answer** (15 marks: 3 marks per part)

a Find the area to the left of $x = \sqrt{y}$ and to the right of $y = x + 10$ between $y = 0$ and $y = 1$.

b Could the area beneath $y = \sqrt{1 - x^4}$ from $x = -1$ to $x = 1$ be smaller than $\frac{3}{2}$? You must *justify* your answer.

c Evaluate $\int_0^{\pi} \cos^4 r dr$.

d Find the critical points of $f(x)$ where $f(x) = \int_2^{(x+1)^3} \cos t dt$.

e Find $\int \frac{x^2}{1+x^6} dx$.

2 **Long Answer** (10 marks)

Consider $f(x) = 7 - 3x$.

a Approximate the area beneath $f(x)$ from $x = 0$ to $x = 2$ by using a Riemann sum with $n = 2$ rectangles and with *right endpoints* as the sample points.

b Is this an underestimate or an overestimate?

c If $n = 100$ were used instead, still with right endpoints as sample points, what would the approximation be?

Give your answer as a *number with two decimals of precision*. You may (or may not) find the identity: $1 + 2 + ... + n = \frac{1}{2}n(n + 1)$ useful.

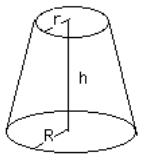
d As $n \to \infty$ these approximations should tend to a limit. What is the value of this limit?

3 **Long Answer** (10 marks)

Find the volume of the described solid: a frustum of a right circular cone with height h, lower base radius R, and top radius r.

See figure 1 for a picture.

Figure 1: Frustum



4 **Long Answer** (10 marks)

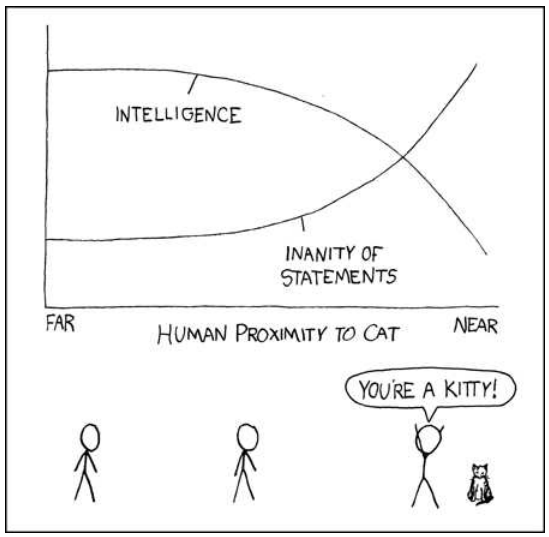
Two cars, A and B, travel along a straight track. At time $t = 0$ seconds they are both at the start line, although traveling with different trajectories. The velocity of car A in metres per second is given by:

$$v_A(t) = \begin{cases} 3t \text{ if } 0 \leq t \leq 10 \\ 30 \text{ if } t > 10 \end{cases} .$$

The velocity of car B in metres per second is given by $v_B(t) = 30 + kt$ where k is a constant. Find the value of k so that both cars are at the same point at $t = 12$ seconds.

5 **Conceptual Question** (5 marks)

State the Funamental Theorem of Calculus (both parts).



Bonus a (1 mark)

Take a moment to appreciate the XKCD comic and draw a smiley face below (1 mark).

Bonus b (1.5 marks)

Suppose a function $f(x)$ satisfies $f(x) = 2 + \int_1^x f(t)dt$. Find $f(7)$.

This page is provided for rough work.

Mark breakdown:

Question 1 (out of 15)	_____
Question 2 (out of 10)	_____
Question 3 (out of 10)	_____
Question 4 (out of 10)	_____
Question 5 (out of 5)	_____
Bonus (out of 2.5)	_____
TOTAL (out of 50)	_____