Be sure that this examination has 13 pages including this cover

The University of British Columbia

Sessional Examinations - April 2010

Mathematics 101

Integral Calculus with Applications to Physical Sciences and Engineering

Closed book examination Time: 2.5 hours

Last Name: _____ **First Name:** _____

Student Number: _____ **Instructor's Name:** _____

Signature: _____ **Section Number:** _____

Rules governing examinations

1. Each candidate must be prepared to produce, upon request, a UBC-card for identification.
2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
3. No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.
4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
   (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
   (b) Speaking or communicating with other candidates.
   (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material form the examination room without permission of the invigilator.
6. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		18
2		18
3		24
4		8
5		8
6		8
7		8
8		4
9		4
Total		100

Marks

[18] **1. Short-Answer Questions.** Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty. Full marks will be given for correct answers placed in the box, but at most 1 mark will be given for incorrect answers. **Simplify your answer as much as possible in this question.**

(a) Evaluate $\int_0^1 (\sqrt{x^3} + (x^2)^{1/3})\, dx$

Answer

(b) Evaluate $\int_0^{\pi} |\cos x|\, dx$. You must simplify your answer *completely.*

Answer

(c) Find a number $b > 0$ such that the function $f(x) = x - 1$ has average value 0 on the interval $[0, b]$.

Answer

Continued on page 3

(d) The first two nonzero terms in the power-series representation in powers of x (i.e. the Maclaurin series) for $\int_0^x \sin(t^2)\ dt$ have the form $ax^3 + bx^7$, for some constants a and b. Find the value of b.

Answer

(e) Find the x-coordinate of the centroid of the region below $y = 1/x^2$ and above the x-axis, from $x = 1$ to $x = 2$.

Answer

(f) Evaluate $\int_{-\infty}^{-1} e^{2x}\ dx$

Answer

Continued on page 4

Full-Solution Problems. In questions 2–9, justify your answers and **show all your work**. If a box is provided, write your final answer there. Unless otherwise indicated, **simplification of answers is not required in these questions.**

[18] **2.** (a) [6] The bounded region that lies between the x-axis and the curve $y = 1 - x^2$ is revolved about the line $y = -2$. Find the volume of the resulting solid of revolution.

Answer

(b) [6] Find the area of the region enclosed by one loop of the polar curve $r = 4\cos 2\theta$.

Answer

Continued on page 5

(c) [6] Find the length of the curve $y = x^2 - \dfrac{\ln x}{8}$, for $1 \leq x \leq 2$.

Answer

Continued on page 6

[24] **3.** Evaluate the following integrals.

(a) [6]

$$\int \frac{x^2 - 9}{x(x^2 + 9)}\, dx$$

Answer

(b) [6]

$$\int \frac{dx}{(4 + x^2)^{3/2}}$$

Answer

Continued on page 7

(c) [6]

$$\int x \tan^{-1} x \; dx$$

(Note: $\tan^{-1} x$ is also denoted $\arctan x$.)

Answer

(d) [6]

$$\int_0^3 x\sqrt{81 - x^4} \; dx$$

Hint: Use a substitution and interpret the result as an area.

Answer

Continued on page 8

[8] **4.** A colony of ants builds an anthill that is in the shape of a cone whose base, at ground level, is a circle of *diameter* 1 ft and whose height is also 1 ft. How much total work, in ft·lbs, is done by the ants in building the anthill? For the density of sand, use the value 150 lb/ft^3.

Answer

Continued on page 9

[8] **5.** An unknown function $f(x)$ has the following values:

$$f(0) = 1, f(1) = 2, f(2) = 0, f(3) = -2, f(4) = -4.$$

(a) [2] Write down the trapezoidal approximation T_4 for $\int_0^4 f(x)\ dx$.

Answer

(b) [2] Write down the Simpson's approximation S_4 for $\int_0^4 f(x)\ dx$.

Answer

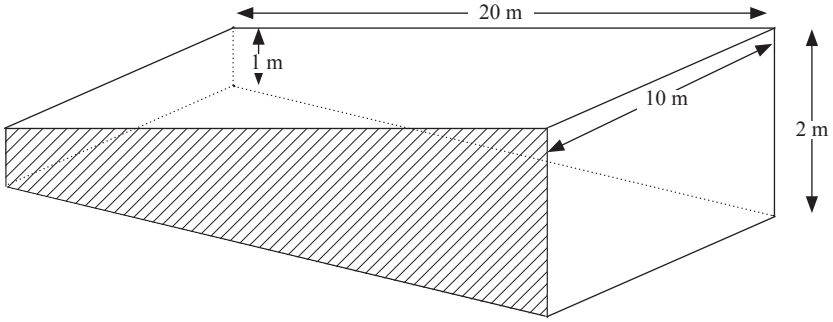
(c) [4] It is known that the fourth derivative $f^{(4)}(x)$ lies between -3 and 2 on the interval $[0, 4]$. What is the largest possible value that $\int_0^4 f(x)\ dx$ could have? You may use the fact that if $|f^{(4)}(x)| \leq K$ on the interval $[a, b]$, then the error in using S_n to approximate $\int_a^b f(x)\ dx$ has absolute value less than or equal to $K(b-a)^5/180n^4$.

Answer

Continued on page 10

[8] **6.** A swimming pool is 10 m wide and 20 m long and its bottom is an inclined plane, the shallow end having depth 1 m and the deep end 2 m. If the pool is full of water, find the hydrostatic force, in Newtons (N), on the long vertical side of the pool that is shaded in the diagram below. For the acceleration due to gravity use the value $g = 9.8$ m/s^2 and for the density of water use the value $\rho = 1000$ kg/m^3.

Answer



Continued on page 11

[8] **7.** Find the solution of the differential equation

$$\frac{dL}{dt} = kL^2 \ln t$$

that satisfies $L(1) = 1$. Here, k is a constant that will appear in your final answer.

Answer

Continued on page 12

[4] **8.** Evaluate $\int_0^1 (6 - 3x^2)\, dx$ by computing the limit of the right-endpoint Riemann sums R_n as $n \to \infty$. You may use the formula $\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$. *No credit will be given for computing this integral using antidifferentiation, although you may use antidifferentiation to check your answer.*

Continued on page 13

[4] **9.** For any real number x, define $g(x) = \int_0^1 (xe^t - t)^2 \, dt$. Find the minimum value of $g(x)$.

Answer

The End