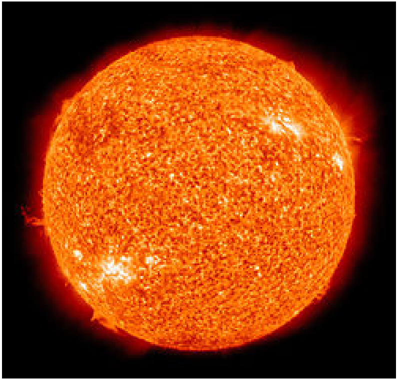
Investigation into the Feasibility and Operation of a Magnetized Target Fusion Reactor, and Qualitative Predictions of Magnetic Field Profile Perturbations Induced by Surface Roughness in Type II Superconductors

Michael Lindstrom, PhD Candidate
Brian Wetton, Supervisor
Michael Ward, Committee Member
Rob Kiefl, Committee Member

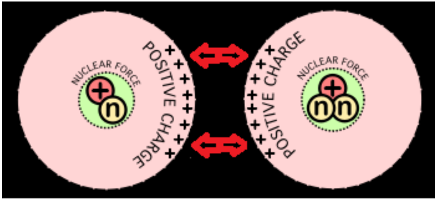
Fusion energy context



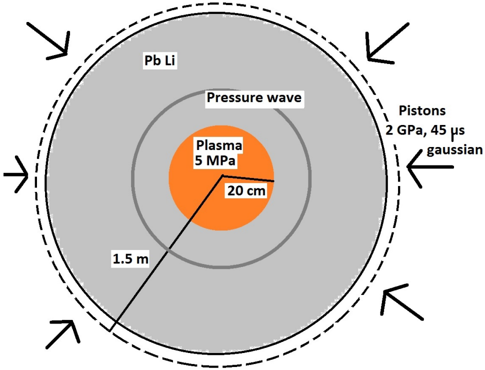
General Fusion (2002-): attempting to produce clean, sustainable fusion energy on earth.

Fusion

- Fusing atomic nuclei yield new nuclei plus energy
- Lawson criterion for energy yield:
  density $\times$ temperature $\times$ time $\geq 4 \times 10^{15}$ cm^{-3} KeV s



General Fusion design



- Magnetized target fusion: magnetically confine plasma with magnetic field, implode in metal cavity

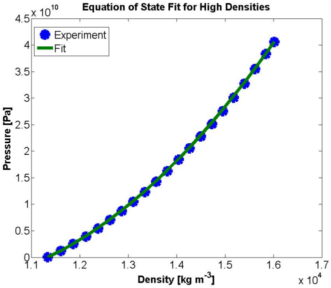
Lead-Lithium

- With density ρ, velocity v, and pressure P:

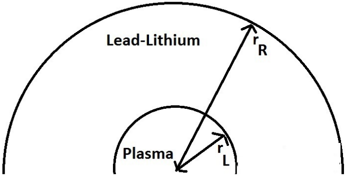
$$\rho_t + \nabla \cdot (\rho v) = 0 \qquad \text{(mass conservation)}$$

$$(\rho v)_t + \nabla \cdot (\rho v \otimes v) + \nabla P = 0 \qquad \text{(momentum conservation)}$$

- Empirical fit to lead experiments $P = P(\rho)$



Pistons, plasma, and general simplifications



- Spherical symmetry
- Pressure: Piston (Gaussian), plasma (gas and magnetic)
- Reversible conditions; equilibrium initialization
- No mixing of plasma and lead-lithium:

$$\frac{\mathrm{d}}{\mathrm{d}t} r_{\text{boundary}}(t) = v(r_{\text{boundary}}(t), t)$$

Overall model

In $r_L(t) < r < r_R(t)$, $t > 0$, dimensionless system has form:

$$\rho_t + \frac{1}{r^2}(r^2 \rho v)_r = 0, \quad (\rho v)_t + p_r + \frac{1}{r^2}(r^2 \rho v^2)_r = 0 \tag{1}$$

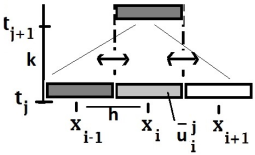
$$p = p(\rho), \quad \frac{\mathrm{d} r_{L,R}}{\mathrm{d}t} = v(r_{L,R}(t), t) \tag{2}$$

$$p(r_L(t), t) = p_L(r_L(t)), \quad p(r_R(t), t) = f(t) \tag{3}$$

$$v(r, 0) = 0, \quad p(r, 0) \text{ constant} \tag{4}$$

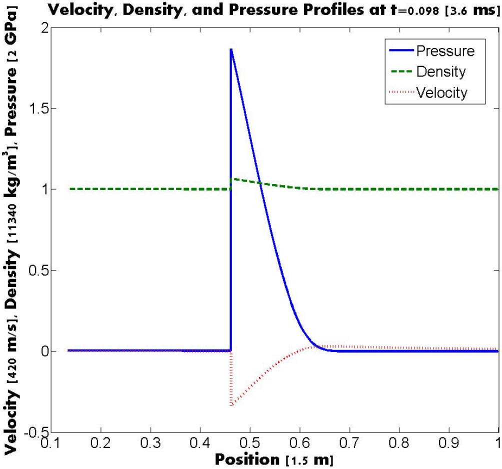
$$r_L(0) \text{ given}, \quad r_R(0) = 1 \tag{5}$$

Finite volume methodology



- Conservation $u_t + (f(u))_x = 0$:
  $\bar{u}_i^{j+1} = \bar{u}_i^j - k\frac{\mathcal{F}^j_{i+1/2} - \mathcal{F}^j_{i-1/2}}{h}$
- $\mathcal{F}$ combination of low/high resolution via limiters
- L^1 convergence: $\int |u_{\mathrm{num}}(x,t) - u_{\mathrm{ex}}(x,t)|\mathrm{d}x = O(h^p)$
- Fixed space domain via coordinate change
- Local linearized systems, approximate Riemann solvers
- Split stepping for geometric sources

Pulse profiles

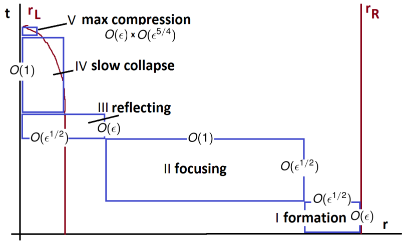


Abridged sensitivity analysis

Table: Min radius R_{min}, Lawson triple product Π_L, impact pressure P_{impact}, initial plasma radius $R_{\text{plasma,0}}$, initial sphere radius $R_{\text{lead,0}}$.

System	R_{min} (cm)	Π_L (10^{15} keV s cm^{-3})
Baseline	3.6	0.52
$R_{\text{plasma,0}} \times 1.1$	5.5	0.25
$P_{\text{impact}} \times 1.1$	3.0	0.64
$R_{\text{lead,0}} \times 1.1$	3.0	0.92
$P_{\text{impact}} \times 2$	1.2	16
$R_{\text{lead,0}} \times 2$	0.84	2.5

Qualitative story and techniques



Matched asymptotics $r_{\min} \sim \frac{b^4\chi^3\mu}{\pi}\epsilon$: reduced pulse time $\epsilon = 0.0126 \ll 1$, sound speed b, radius χ, pressure μ

- I formation: Riemann invariants
- II focusing: velocity potential
- III reflection: boundary conditions imply long-term velocity
- IV/V compression: velocity radially dependent

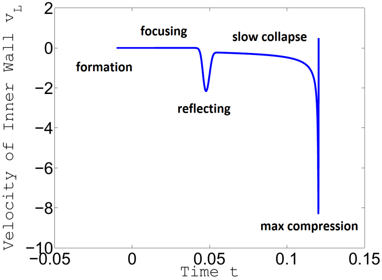
Minimum radius

- Dimensional minimum radius

$$R_{\min} \approx \frac{C_s^4 P_{\text{plasma},0} R_{\text{plasma},0}^7 \varrho_0^3}{\pi P_{\text{impact}}^4 R_{\text{lead},0}^4 t_0^2} = 1.6 \text{ cm}$$

Symbol	Meaning	Symbol	Meaning
C_s	lead sound speed	$P_{\text{plasma},0}$	initial pressure
$R_{\text{plasma},0}$	initial plasma radius	ϱ_0	lead density
P_{impact}	piston pressure	$R_{\text{lead},0}$	initial lead radius
t_0	impulse time scale		

Key insights



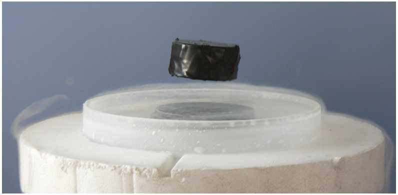
- Almost all input energy reflected:

$$E_{\text{input}} \sim \frac{\sqrt{8\pi^3}}{b}\epsilon^{3/2}, \quad E_{\text{compression}} \sim \frac{4\pi^2}{b^4\chi^3}\epsilon^{5/2}$$

Results and future work

- Results:
  - energy yield may be within reach
  - larger outer sphere radius and impact pressure noteworthy
- Future directions:
  - more physics
  - effects of imperfect spherical symmetry
  - more precise assessment of design

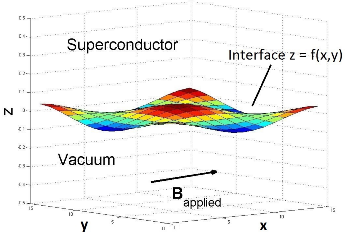
Superconductor roughness context

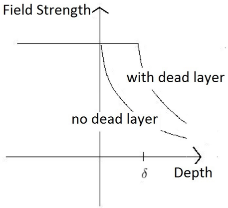


Superconductors expel magnetic fields, some unresolved questions that arise in comparing theory to experiment.

Overview

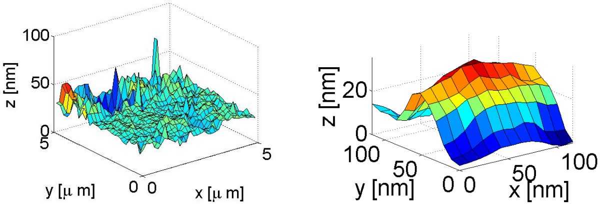
- Superconductors:
  - cold enough - no resistance, expel magnetic fields
  - YBCO studied experimentally, unexpected field profiles
- London Model:
  - field decays from applied value exponentially with length scale λ with flat surface
  - experiments find dead layer: could roughness cause this?



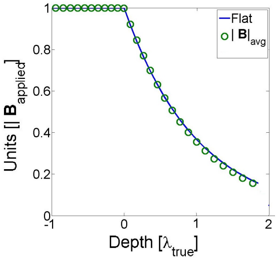


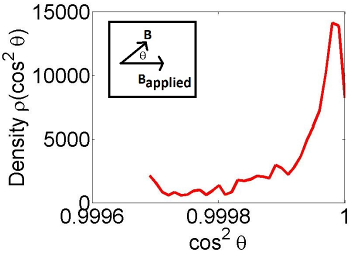
Methodology

- Use real AFM surface data to study how fitting parameters affected



Results





- λ may be underestimated, almost no dead layer: best fitting (λ, δ) are $(0.956\lambda_{\text{true}}, 0.016\lambda_{\text{true}})$
- Minute change in field orientation

Future work

- Extend simulations to spatially varying order-parameters
- Consider anisotropic superconductors

Thanks to...

Thanks to the following people for helpful discussions, collaboration, and support:

- My committee: Brian Wetton, Michael Ward, Rob Kiefl (UBC)
- Alex Fang (UBC)
- Sandra Barsky and Aaron Froese (General Fusion)
- George Bluman (UBC)
- Randy LeVeque (University of Washington)
- My family, friends, colleagues, and fellow graduate students